Venn Diagram
&
DeMorgan's Law

What is a **Venn Diagram**?

Venn Diagram is frequently used by statisticians to display relationships between events in a sample space.

How do we construct a **Venn Diagram**?

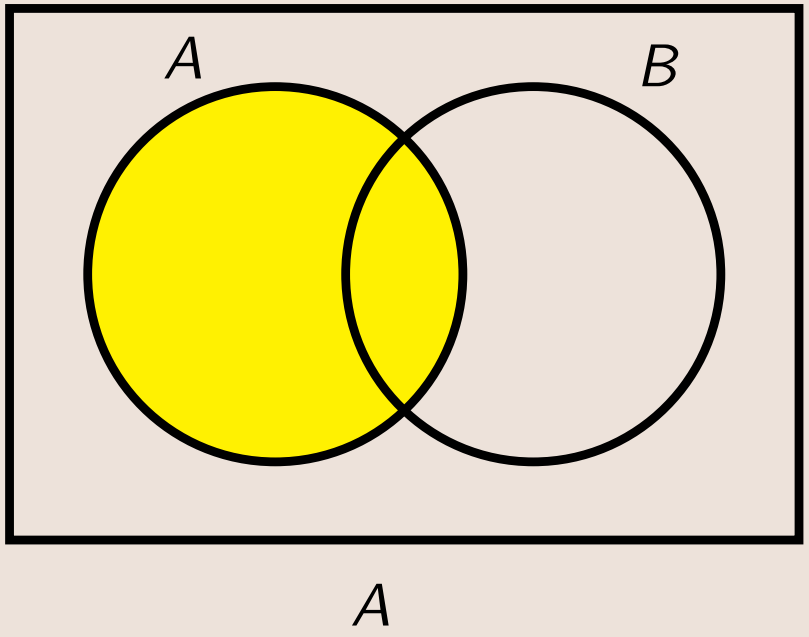
When working with probabilities, the following criteria for constructing **Venn Diagram** should be followed,

- the sample space is represented by a rectangle.
- events within the sample space are often displayed by circles, clearly labeled by corresponding probabilities.
- the sum of all probabilities within the rectangle is equal to 1.

Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event A.

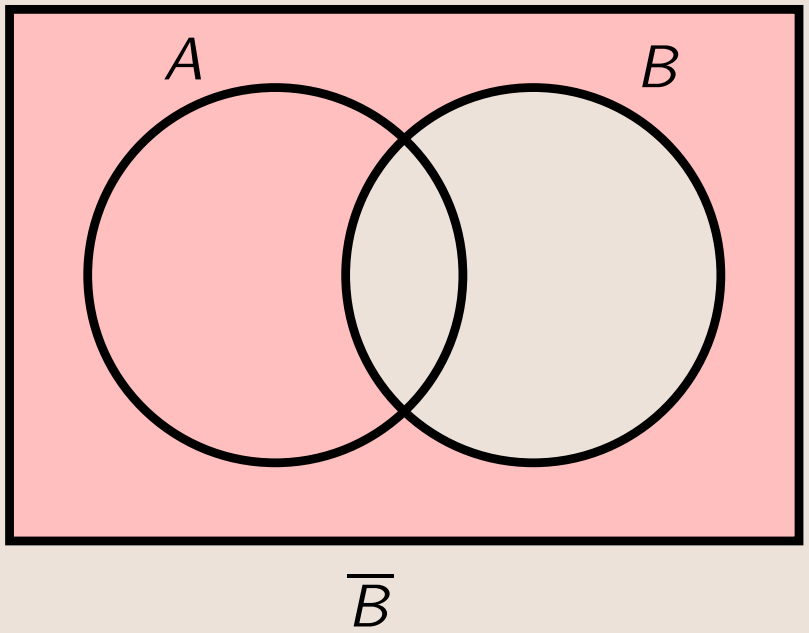
Solution:



Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event $\overline{B}$.

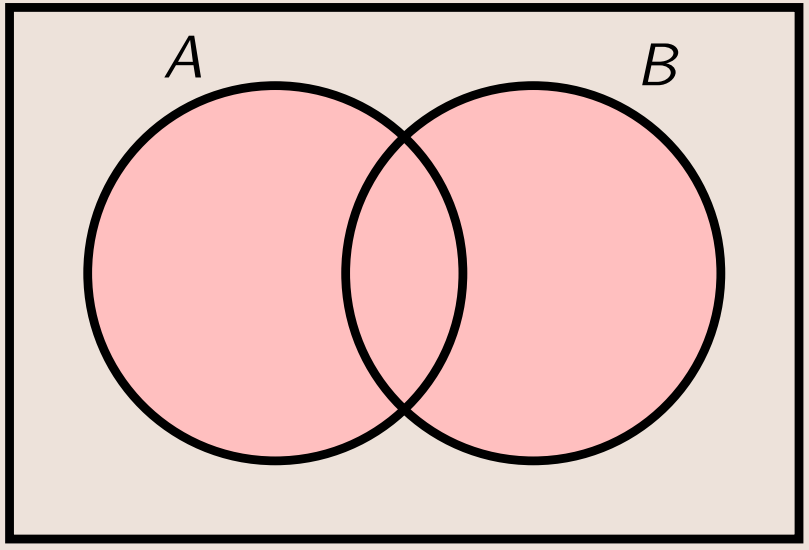
Solution:



Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event A or B.

Solution:

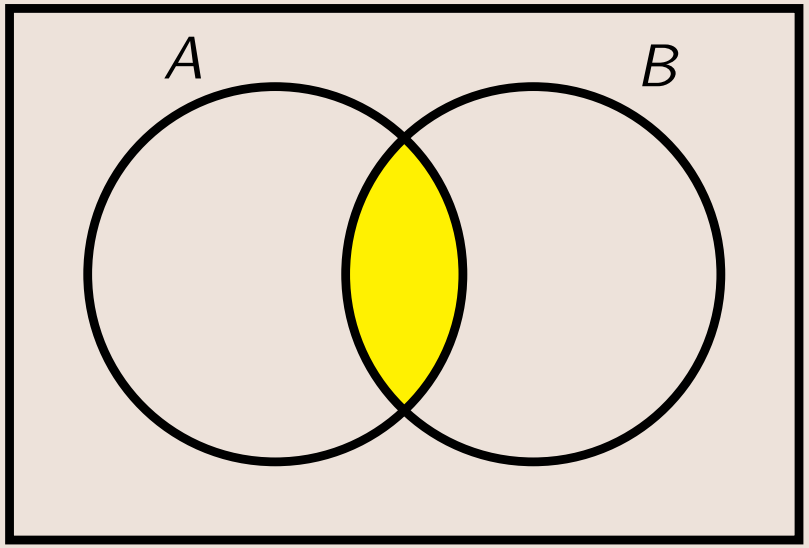


A or B

Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event A and B.

Solution:

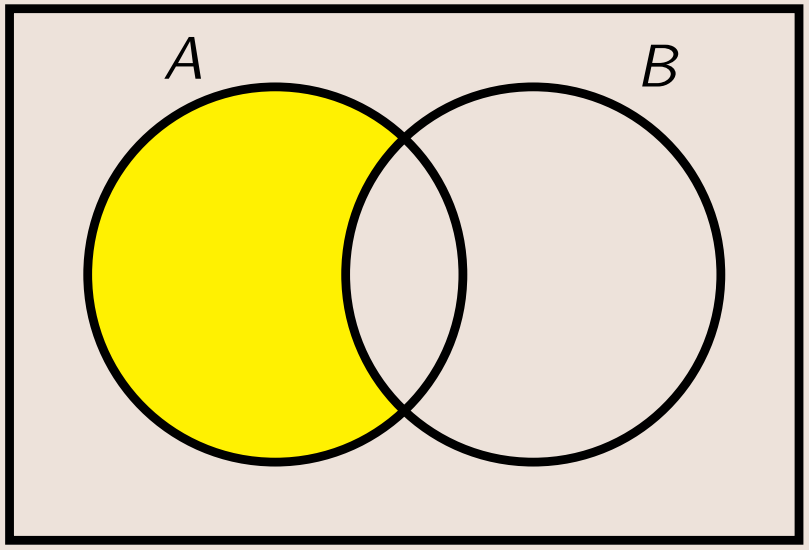


A and B

Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event A only.

Solution:

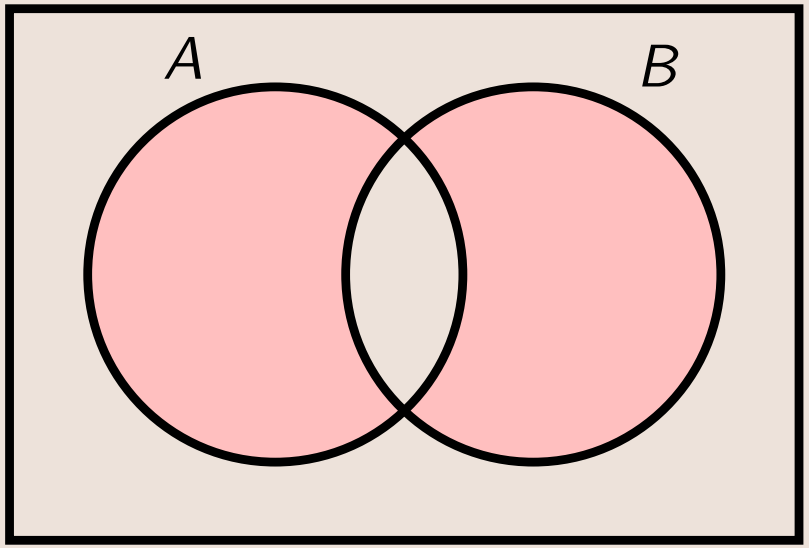


A only

Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event A only or B only.

Solution:

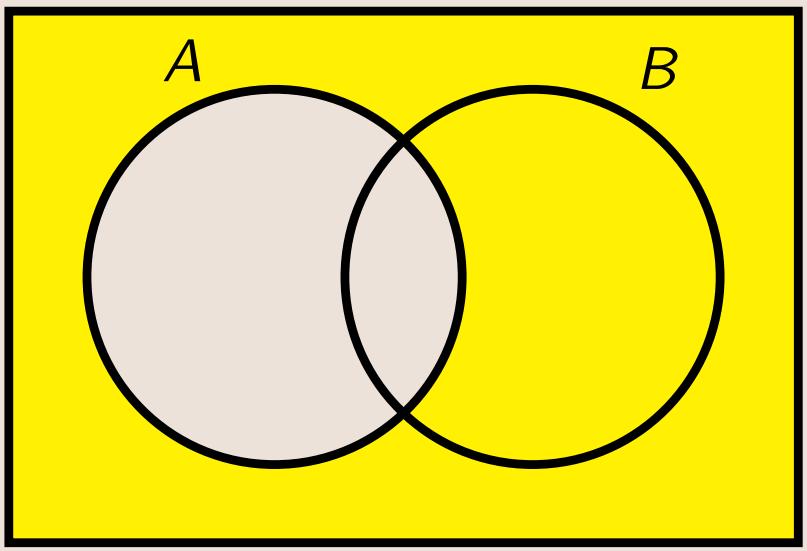


A only or B only

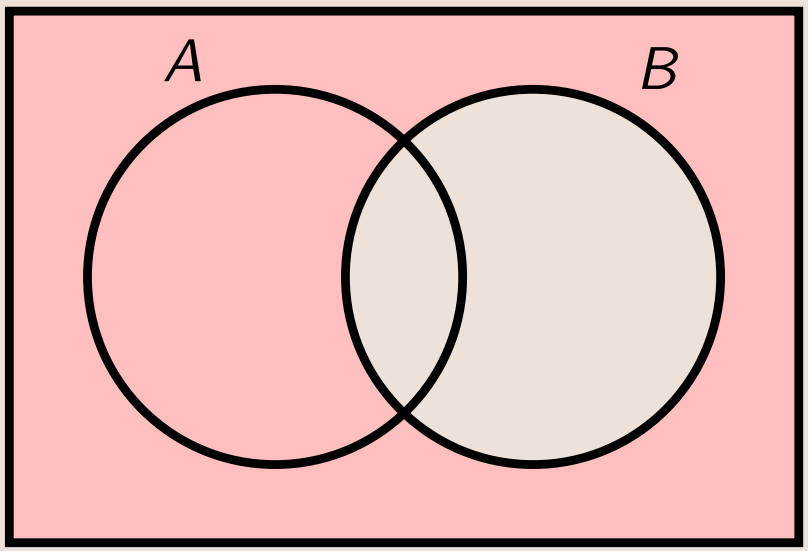
Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event $\overline{A}$ or $\overline{B}$.

Solution:



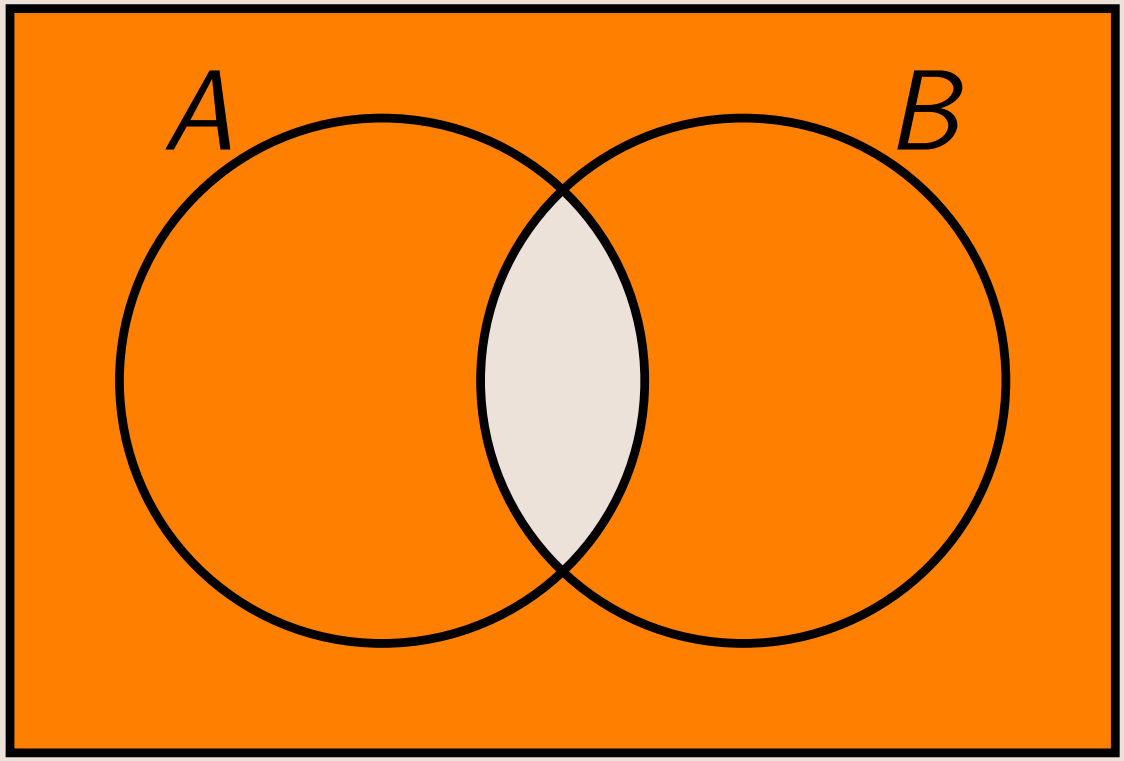
$\overline{A}$



$\overline{B}$

Solution Continued:

Now if we superimpose these two Venn diagrams, the only region that is not shaded is the overlap of these two events.
When working with **OR**, we take all the regions that are shaded at least once.

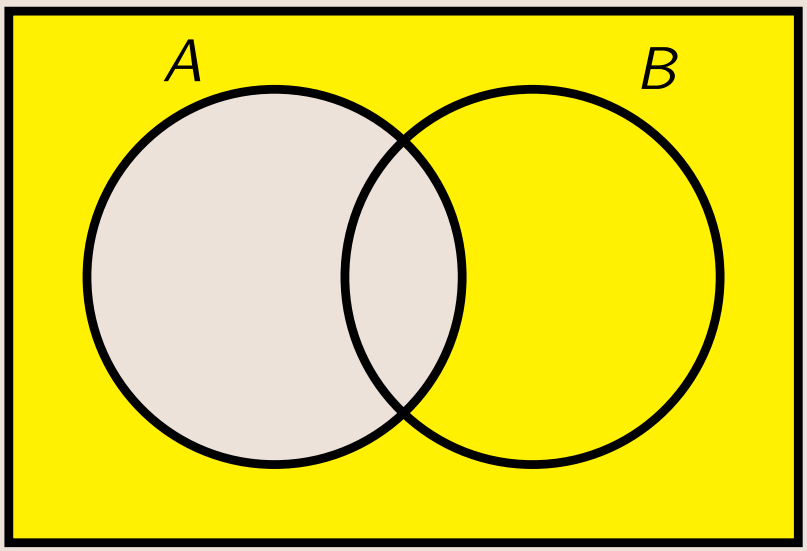


$\overline{A}$ or $\overline{B}$

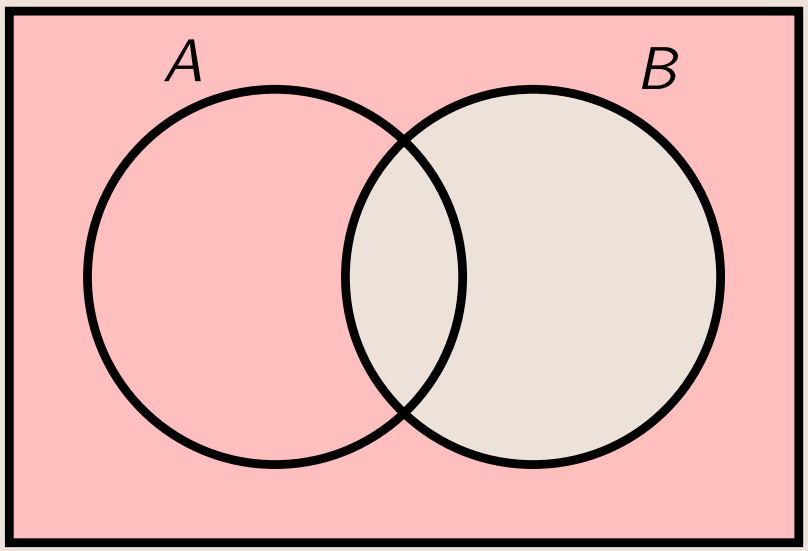
Example:

Construct a Venn Diagram with two events A and B, then clearly shade the event $\overline{A}$ and $\overline{B}$.

Solution:



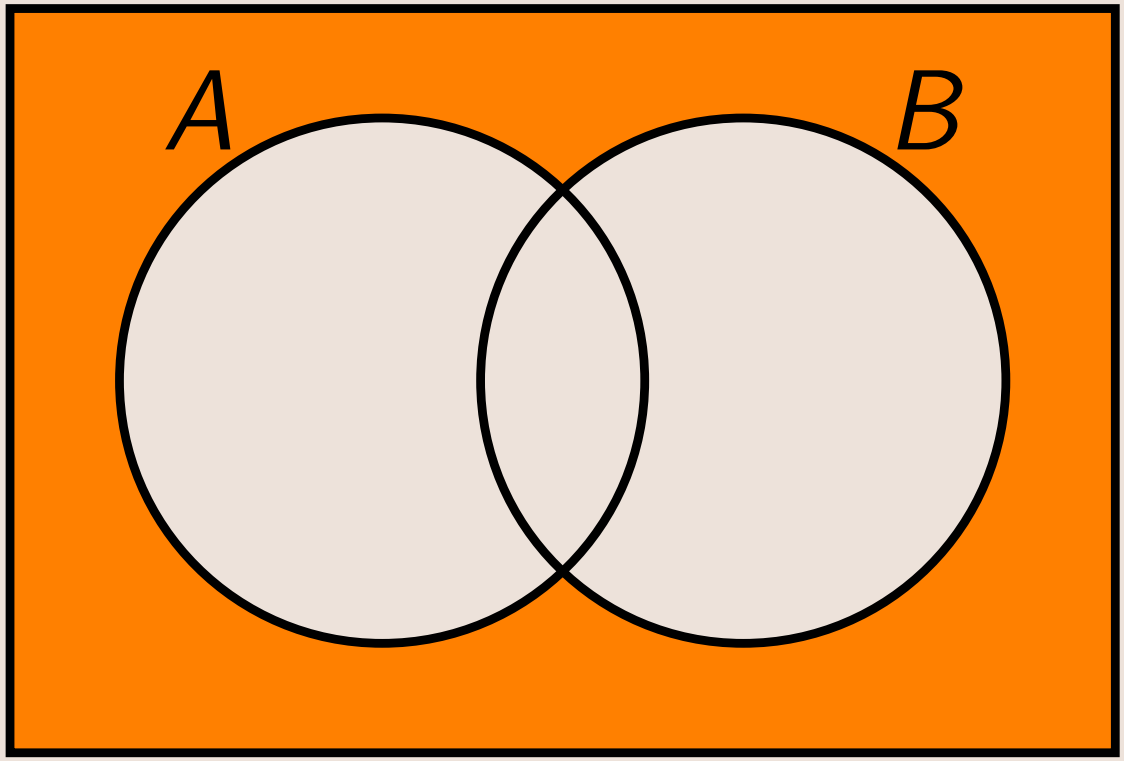
$\overline{A}$



$\overline{B}$

Solution Continued:

Now if we superimpose these two Venn diagrams, the only common shaded region is the outside of both events.
When working with **AND**, we take only the commonly shaded region.



$\overline{A}$ and $\overline{B}$

What is **DeMorgan's Law**?

DeMorgan's Law shows a relationship between the union(**OR**) and the intersection(**AND**) of the complement of two events.

What are the results of **DeMorgan's Law**?

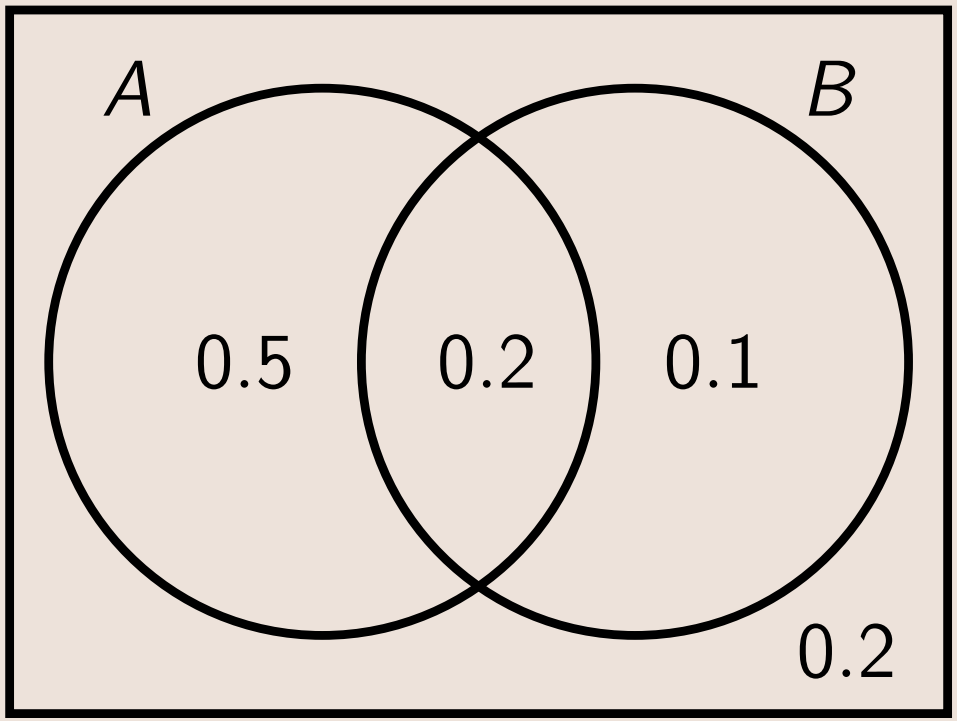
In **Mathematical Notation**, it says

- $P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B})$
- $P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B})$

Example:

Given: $P(A) = 0.7, P(B) = 0.3,$ and $P(A \text{ and } B) = 0.2,$
Construct the Venn Diagram using the given information.

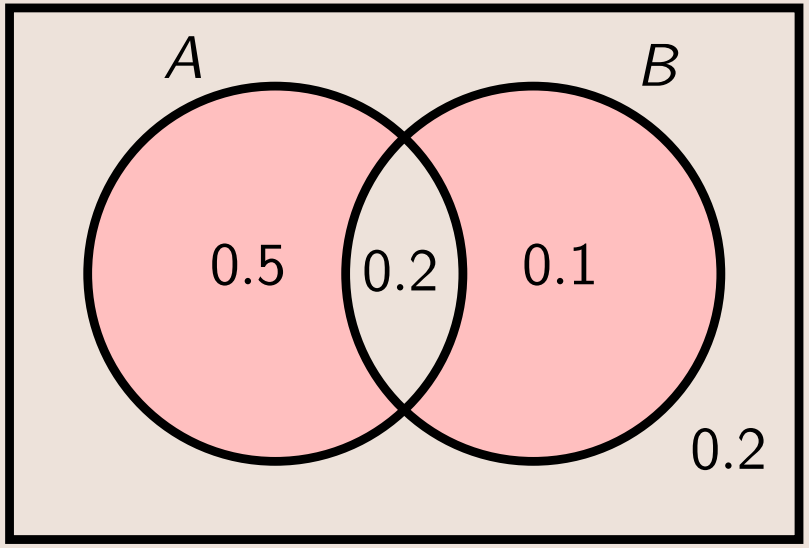
Solution:



Example:

Use the last example to find $P(A \text{ only or } B \text{ only})$.

Solution:



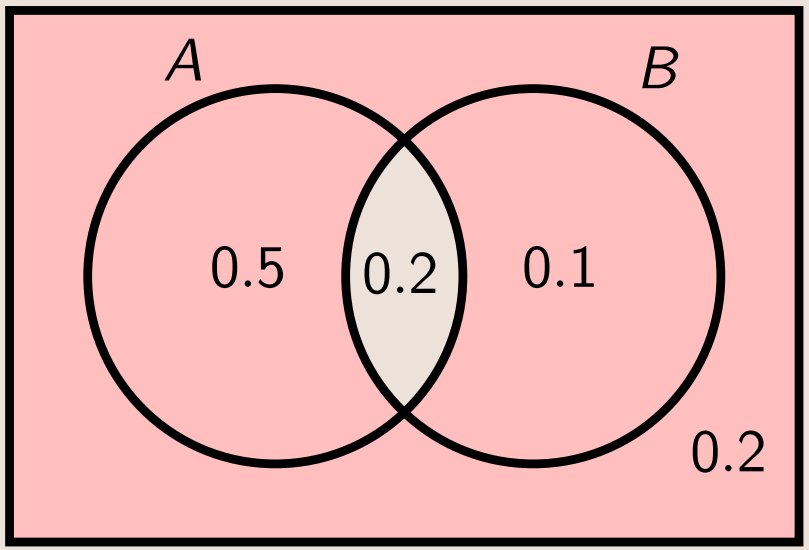
A only or B only

$P(A \text{ only or } B \text{ only}) = 0.5 + 0.1 = 0.6$

Example:

Use the last example to find $P(\overline{A} \text{ or } \overline{B})$.

Solution:



$\overline{A}$ or $\overline{B}$

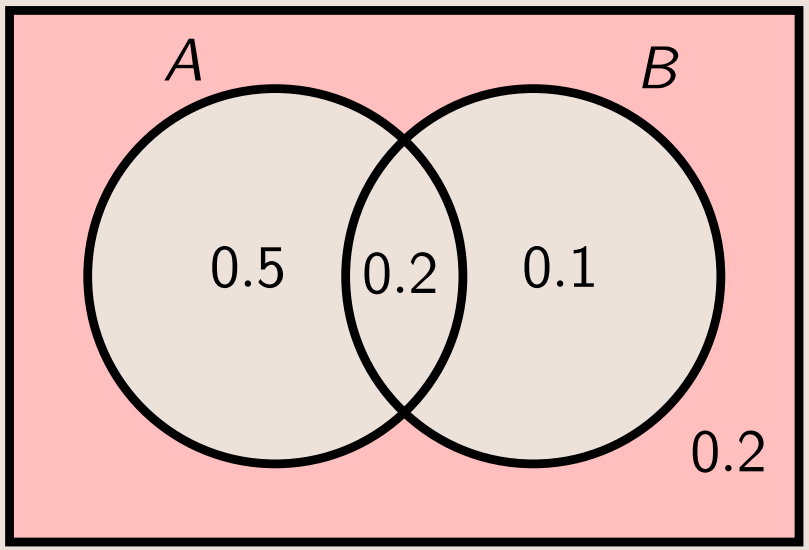
Using DeMorgan's Law, we get

$$P(\overline{A} \text{ or } \overline{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - 0.2 = 0.8$$

Example:

Use the last example to find $P(\overline{A} \text{ and } \overline{B})$.

Solution:



$\overline{A}$ and $\overline{B}$

Using DeMorgan's Law, we get

$$P(\overline{A} \text{ and } \overline{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - 0.8 = 0.2$$